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Guessing examples with the package DalgGuessing from https://github.com/T3gu1a/D-algebraic-Guessing

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> with(DalgGuessing)

[DalgFunGuess, modDalgFunGuess] **(1)**

> with(CodeTools, CPUTime) #CPUTime is often used to display the CPU time of computations

[CPUTime] **(2)**

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DalgFunGuess

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Example 1: $\zeta(2n+2), n=0, 1, 2 \ldots$

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> L := [seq(Zeta(2·j + 2), j = 0 ..12)] #13 terms

$$L := \left[\frac{\pi^2}{6}, \frac{\pi^4}{90}, \frac{\pi^6}{945}, \frac{\pi^8}{9450}, \frac{\pi^{10}}{93555}, \frac{691\,\pi^{12}}{638512875}, \frac{2\,\pi^{14}}{18243225}, \frac{3617\,\pi^{16}}{325641566250}, \frac{43867\,\pi^{18}}{38979295480125}, \frac{174611\,\pi^{20}}{1531329465290625}, \frac{155366\,\pi^{22}}{13447856940643125}, \frac{236364091\,\pi^{24}}{201919571963756521875}, \frac{1315862\,\pi^{26}}{11094481976030578125}\right] \quad \textbf{(1.1.1)}$$

> DalgFunGuess(L, degADE = 2, degPoly = 1, startfromord = 2)

$$-\_C\,y(x)^2 + \frac{5\,\_C\left(\frac{d}{dx}\,y(x)\right)}{2} - 2\,\_C\,x\left(\frac{d}{dx}\,y(x)\right)y(x) + \_C\,x\left(\frac{d^2}{dx^2}\,y(x)\right) = 0 \quad \textbf{(1.1.2)}$$

> DalgFunGuess(L[1 ..9], degADE = 2, degPoly = 1, startfromord = 2, allPolyDeg = true) #9 terms

$$-\_C\,y(x)^2 + \frac{5\,\_C\left(\frac{d}{dx}\,y(x)\right)}{2} - 2\,\_C\,x\left(\frac{d}{dx}\,y(x)\right)y(x) + \_C\,x\left(\frac{d^2}{dx^2}\,y(x)\right) = 0 \quad \textbf{(1.1.3)}$$

> CPUTime(DalgFunGuess(L[1 ..7], degADE = 2, degPoly = 1, startfromord = 2,

allPolyDeg = true, sparsity = 1/2)) #7 **terms**

$$0.266, -\_C\, y(x)^2 + \frac{5\,\_C \left(\frac{d}{dx} y(x)\right)}{2} - 2\,\_C\, x \left(\frac{d}{dx} y(x)\right) y(x) + \_C\, x \left(\frac{d^2}{dx^2} y(x)\right) = 0 \quad (1.1.4)$$

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Example 2: Gregory coefficients $\frac{x}{\ln(1+x)} = 1 + \sum_{n=0}^{\infty} G_n x^n$

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```
> f := (x/log(1 + x) - 1)/x:
> L := PolynomialTools:-CoefficientList(convert(series(f, x, 21), polynom), x)
```

$$L := \left[\frac{1}{2}, -\frac{1}{12}, \frac{1}{24}, -\frac{19}{720}, \frac{3}{160}, -\frac{863}{60480}, \frac{275}{24192}, -\frac{33953}{3628800}, \frac{8183}{1036800}, -\frac{3250433}{479001600}, \frac{4671}{788480}, -\frac{13695779093}{2615348736000}, \frac{2224234463}{475517952000}, -\frac{132282840127}{31384184832000}, \frac{2639651053}{689762304000}, -\frac{111956703448001}{32011868528640000}, \frac{50188465}{15613165568}, -\frac{2334028946344463}{786014494949376000}, \frac{301124035185049}{109285437800448000}\right] \quad (1.2.1)$$

```
> numelems(L)
```

$$19 \quad (1.2.2)$$

```
> DalgFunGuess(L, degADE = 2, degPoly = 2, startfromord = 2) #19 terms
```

$$\_C\, y(x)^2 + 2\,\_C\, x \left(\frac{d}{dx} y(x)\right) y(x) + (2\,\_C\, x + 3\,\_C) \left(\frac{d}{dx} y(x)\right) + (\_C\, x^2 + \_C\, x) \left(\frac{d^2}{dx^2} y(x)\right) = 0 \quad (1.2.3)$$

```
> CPUTime(DalgFunGuess(L[1 ..11], degADE = 2, degPoly = 2, startfromord = 2,
      allPolyDeg = true))#11 terms
```

$$1.328, \_C\, y(x)^2 + 2\,\_C\, x \left(\frac{d}{dx} y(x)\right) y(x) + (2\,\_C\, x + 3\,\_C) \left(\frac{d}{dx} y(x)\right) + (\_C\, x^2 + \_C\, x) \left(\frac{d^2}{dx^2} y(x)\right) = 0 \quad (1.2.4)$$

```
> CPUTime(DalgFunGuess(L[1 ..9], degADE = 2, degPoly = 2, startfromord = 2,
      allPolyDeg = true, sparsity = 1/2)) #9 terms and wrong guess
```

$$28.656, \frac{298462802951078\,\_C\,x^2\,y(x)^2}{6212024644107945} + \left(\frac{757941517751174128}{414134976273863}\,\_C\,x + \frac{347399047730767608}{2070674881369315}\,\_C\,x^2\right)\left(\frac{d}{dx}\,y(x)\right)^2 + \left(-\frac{912507173808295 04}{2070674881369315}\,\_C\,x - \frac{38269387337136158}{2070674881369315}\,\_C\,x^2\right)\left(\frac{d}{dx}\,y(x)\right)y(x) + \left(\frac{77205906414442392}{2070674881369315}\,\_C\,x + \frac{33806828819378548}{6212024644107945}\,\_C\,x^2\right)\left(\frac{d}{dx}\,y(x)\right) + \left(-\frac{69769792316}{414134976273863}\,\_C\,x + \_C\,x^2\right)\left(\frac{d^2}{dx^2}\,y(x)\right) = 0 \quad (1.2.5)$$

> $CPUTime\left(DalgFunGuess\left(L[1..10],\, degADE = 2,\, degPoly = 2,\, startfromord = 2,\, allPolyDeg = true,\, sparsity = \frac{1}{2}\right)\right)$ #10 terms

$$2.407, \_C\,y(x)^2 + 2\,\_C\,x\left(\frac{d}{dx}\,y(x)\right)y(x) + (2\,\_C\,x + 3\,\_C)\left(\frac{d}{dx}\,y(x)\right) + (\_C\,x^2 + \_C\,x)\left(\frac{d^2}{dx^2}\,y(x)\right) = 0 \quad (1.2.6)$$

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Example 3: Dividing D-algebraic functions:

An instance of finding an algebraic differential equation satisfied by ALL $\frac{f}{g}$, where $p(f) = q(g) = 0$

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```
#The differential polynomials
p:=diff(y(x), x) - x*y(x)^2:
q:=-diff(z(x), x) + z(x) + x + 1:
sys := {p=0,q=0}: #Note: it is better to treat both ADEs separately as they are independent

#symbolic initial conditions
InitialConds:= {y(0) = theta, z(0) = beta}:

#Order is a global variable for truncating series solutions
Order :=19:

#Computing truncations of f and g of order Order
sol := dsolve(sys union InitialConds, {y(x), z(x)}, series):

# Extract the truncations
f_series := eval(y(x), sol):
g_series := eval(z(x), sol):

# Convert to polynomial
h_series := series(f_series/g_series, x, Order):
h_poly := convert(h_series, polynom):

# List ot coefficients
L:=PolynomialTools:-CoefficientList(h_poly,x):
```

> $simplify(L[1..5])$ #5 first terms

$$\left[\frac{\theta}{\beta}, -\frac{\theta\,(1+\beta)}{\beta^2}, \frac{(2 + (\theta + 1)\,\beta^2 + 2\,\beta)\,\theta}{2\,\beta^3}, \right. \quad (1.3.1)$$

$$-\frac{\left(2+\left(\theta+\frac{1}{3}\right)\beta^3+\left(\theta+\frac{2}{3}\right)\beta^2+2\beta\right)\theta}{2\beta^4},$$

$$\left.\frac{\left(4+\left(\theta^2+\theta+\frac{1}{6}\right)\beta^4+\left(2\theta-\frac{1}{3}\right)\beta^3+\left(2\theta+\frac{2}{3}\right)\beta^2+4\beta\right)\theta}{4\beta^5}\right]$$

> *numelems*(*L*)

19 **(1.3.2)**

> *t*, *ADE* := *CPUTime*(*DalgFunGuess*(*L*, *degADE* = 2, *degPoly* = 5, *startfromord* = 1, *inputConstants* = {beta, theta}))*#19 terms*

$$t, ADE := 1.672, \left(-\frac{2\_C}{\theta^2}-\frac{2\_C x}{\theta^2}+\frac{2\_C x^2}{\theta}+\frac{2\_C x^3}{\theta}-\frac{\_C x^4}{2}-\frac{\_C x^5}{2}\right)y(x)^2+\left(-\frac{2\_C}{\theta}+2\_C x+\_C x^2\right)y(x)+\left(-\frac{2\_C}{\theta}+\_C x^2\right)\left(\frac{d}{dx}y(x)\right)=0 \quad \textbf{(1.3.3)}$$

> *CPUTime*(*DalgFunGuess*(*L*[1..13], *degADE* = 2, *degPoly* = 5, *startfromord* = 1, *inputConstants* = {beta, theta}, *allPolyDeg* = *true*))*#13 terms*

$$6.250, \left(-\frac{2\_C}{\theta^2}-\frac{2\_C x}{\theta^2}+\frac{2\_C x^2}{\theta}+\frac{2\_C x^3}{\theta}-\frac{\_C x^4}{2}-\frac{\_C x^5}{2}\right)y(x)^2+\left(-\frac{2\_C}{\theta}+2\_C x+\_C x^2\right)y(x)+\left(-\frac{2\_C}{\theta}+\_C x^2\right)\left(\frac{d}{dx}y(x)\right)=0 \quad \textbf{(1.3.4)}$$

> *CPUTime*(*DalgFunGuess*(*L*[1..12], *degADE* = 2, *degPoly* = 5, *startfromord* = 1, *inputConstants* = {beta, theta}, *allPolyDeg* = *true*, *sparsity* = $\frac{1}{3}$, *maxIteration* = 50))

#12 terms, would only succeed with luck

12.359, *FAIL* **(1.3.5)**

Eliminating θ

> $\rho_1 := subs\left(\left[\_C = 2\cdot\theta^2, y(x) = y, diff(y(x), x) = y1\right], lhs(ADE)\right)$

$$\rho_1 := \left(-\theta^2 x^5-\theta^2 x^4+4\theta x^3+4\theta x^2-4x-4\right)y^2+\left(2\theta^2 x^2+4\theta^2 x-4\theta\right)y+\left(2\theta^2 x^2-4\theta\right)y1 \quad \textbf{(1.3.6)}$$

> $\rho_2 := subs\left(\left[\_C = 2\cdot\theta^2, y(x) = y, diff(y(x), x) = y1, diff(y(x), x, x) = y2\right], diff(lhs(ADE), x)\right)$

$$\rho_2 := \left(-5\theta^2 x^4-4\theta^2 x^3+12\theta x^2+8\theta x-4\right)y^2+2\left(-\theta^2 x^5-\theta^2 x^4+4\theta x^3+4\theta x^2-4x-4\right)y\,y1+\left(4\theta^2 x+4\theta^2\right)y+\left(2\theta^2 x^2+4\theta^2 x-4\theta\right)y1 \quad \textbf{(1.3.7)}$$

$$+ 4\theta^2 x\, y1 + \left(2\theta^2 x^2 - 4\theta\right) y2$$

> rho := $resultant(\rho_1, \rho_2, \text{theta})$

$$\rho := -1024x^6y^7 - 3072x^5y^7 - 3072x^4y^7 - 256x^4y^6 - 1536x^4y^5y1 + 1024x^4y^5y2 - 2048x^4y^4y1^2 - 1024x^3y^7 - 1536x^3y^6 - 4096x^3y^5y1 + 2048x^3y^5y2 - 4096x^3y^4y1^2 - 1280x^2y^6 - 3072x^2y^5y1 + 1024x^2y^5y2 - 2048x^2y^4y1^2 - 256x^2y^4y1 + 256x^2y^4y2 - 1024x^2y^3y1^2 + 768x^2y^3y1\,y2 - 256x^2y^3y2^2 - 1536x^2y^2y1^3 + 1024x^2y^2y1^2y2 - 1024x^2y\,y1^4 - 512x\,y^5y1 - 256x\,y^5 - 1280x\,y^4y1 + 768x\,y^4y2 - 2816x\,y^3y1^2 + 1280x\,y^3y1\,y2 - 256x\,y^3y2^2 - 2560x\,y^2y1^3 + 1024x\,y^2y1^2y2 - 1024x\,y\,y1^4 + 256y^6 - 256y^5 - 768y^4y1 + 256y^4y2 - 1024y^3y1^2 + 256y^3y1\,y2 - 512y^2y1^3 \quad \textbf{(1.3.8)}$$

> $factor(\text{rho})$

$$-256y\left(4x^6y^6 + 12x^5y^6 + 12x^4y^6 + x^4y^5 + 6x^4y^4y1 - 4x^4y^4y2 + 8x^4y^3y1^2 + 4x^3y^6 + 6x^3y^5 + 16x^3y^4y1 - 8x^3y^4y2 + 16x^3y^3y1^2 + 5x^2y^5 + 12x^2y^4y1 - 4x^2y^4y2 + 8x^2y^3y1^2 + x^2y^3y1 - x^2y^3y2 + 4x^2y^2y1^2 - 3x^2y^2y1\,y2 + x^2y^2y2^2 + 6x^2y\,y1^3 - 4x^2y\,y1^2y2 + 4x^2y1^4 + 2x\,y^4y1 + y^4x + 5x\,y^3y1 - 3x\,y^3y2 + 11x\,y^2y1^2 - 5x\,y^2y1\,y2 + x\,y^2y2^2 + 10x\,y\,y1^3 - 4x\,y\,y1^2y2 + 4x\,y1^4 - y^5 + y^4 + 3y^3y1 - y^3y2 + 4y^2y1^2 - y^2y1\,y2 + 2y\,y1^3\right) \quad \textbf{(1.3.9)}$$

> rho := $(collect(op(3, \textbf{(1.3.9)}), [y, y1, y2], distributed))$

$$\rho := \left(4x^6 + 12x^5 + 12x^4 + 4x^3\right)y^6 + \left(x^4 + 6x^3 + 5x^2 - 1\right)y^5 + \left(6x^4 + 16x^3 + 12x^2 + 2x\right)y^4y1 + \left(-4x^4 - 8x^3 - 4x^2\right)y^4y2 + (1 + x)y^4 + \left(8x^4 + 16x^3 + 8x^2\right)y^3y1^2 + \left(x^2 + 5x + 3\right)y^3y1 + \left(-x^2 - 3x - 1\right)y^3y2 + \left(4x^2 + 11x + 4\right)y^2y1^2 + \left(-3x^2 - 5x - 1\right)y^2y1\,y2 + \left(x^2 + x\right)y^2y2^2 + \left(6x^2 + 10x + 2\right)y\,y1^3 + \left(-4x^2 - 4x\right)y\,y1^2y2 + \left(4x^2 + 4x\right)y1^4 \quad \textbf{(1.3.10)}$$

> $ADE := subs([y = y(x), y1 = diff(y(x), x), y2 = diff(y(x), x, x)], \text{rho})$

$$ADE := \left(4x^6 + 12x^5 + 12x^4 + 4x^3\right)y(x)^6 + \left(x^4 + 6x^3 + 5x^2 - 1\right)y(x)^5 + \left(6x^4 + 16x^3 + 12x^2 + 2x\right)y(x)^4\left(\frac{d}{dx}y(x)\right) + \left(-4x^4 - 8x^3 - 4x^2\right)y(x)^4\left(\frac{d^2}{dx^2}y(x)\right) + (1 + x)y(x)^4 + \left(8x^4 + 16x^3 + 8x^2\right)y(x)^3\left(\frac{d}{dx}y(x)\right)^2 + \left(x^2 + 5x + 3\right)y(x)^3\left(\frac{d}{dx}y(x)\right) + \left(-x^2 - 3x - 1\right)y(x)^3\left(\frac{d^2}{dx^2}y(x)\right) + \left(4x^2 + 11x + 4\right)y(x)^2\left(\frac{d}{dx}y(x)\right)^2 + \left(-3x^2 - 5x - 1\right)y(x)^2\left(\frac{d}{dx}y(x)\right)\left(\frac{d^2}{dx^2}y(x)\right) + \left(x^2 + x\right)y(x)^2\left(\frac{d^2}{dx^2}y(x)\right)^2 + \left(6x^2 + 10x + 2\right)y(x)\left(\frac{d}{dx}y(x)\right)^3 + (\quad \textbf{(1.3.11)}$$

$$-4x^2-4x)\, y(x) \left(\frac{d}{dx} y(x)\right)^2 \left(\frac{d^2}{dx^2} y(x)\right) + (4x^2+4x)\left(\frac{d}{dx} y(x)\right)^4$$

NLDE finds the same ADE from p and q.

> *CPUTime*(*NLDE:-arithmeticDalg*([$p=0, q=0$], [$y(x), z(x)$], $w=\frac{y}{z}$))

$$2.266, (4x^6+12x^5+12x^4+4x^3)\, w(x)^6 + (x^4+6x^3+5x^2-1)\, w(x)^5 + (6x^4 + 16x^3+12x^2+2x)\, w(x)^4 \left(\frac{d}{dx} w(x)\right) + (-4x^4-8x^3-4x^2)\left(\frac{d^2}{dx^2} w(x)\right) w(x)^4 + (x+1)\, w(x)^4 + (8x^4+16x^3+8x^2)\, w(x)^3 \left(\frac{d}{dx} w(x)\right)^2 + (x^2+5x+3)\, w(x)^3 \left(\frac{d}{dx} w(x)\right) + (-x^2-3x-1)\, w(x)^3 \left(\frac{d^2}{dx^2} w(x)\right) + (4x^2+11x+4)\, w(x)^2 \left(\frac{d}{dx} w(x)\right)^2 + (-3x^2-5x-1)\, w(x)^2 \left(\frac{d}{dx} w(x)\right)\left(\frac{d^2}{dx^2} w(x)\right) + (x^2+x)\, w(x)^2 \left(\frac{d^2}{dx^2} w(x)\right)^2 + (6x^2+10x + 2)\, w(x) \left(\frac{d}{dx} w(x)\right)^3 + (-4x^2-4x)\, w(x)\left(\frac{d}{dx} w(x)\right)^2 \left(\frac{d^2}{dx^2} w(x)\right) + (4x^2+4x)\left(\frac{d}{dx} w(x)\right)^4 = 0 \quad \textbf{(1.3.12)}$$

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Example 4: ADE for $e^{e^{e^x}-e}$

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> $f := \exp(\exp(\exp(x)) - \exp(1))$:

> $L :=$ *PolynomialTools:-CoefficientList*(*convert*(*series*($f, x, 80$), *polynom*), x) :

> *numelems*(L)

$$80 \quad \textbf{(1.4.1)}$$

> $L[1..5]$

$$\left[1, e, e + \frac{(e)^2}{2}, \frac{5\,e}{6} + (e)^2 + \frac{(e)^3}{6}, \frac{5\,e}{8} + \frac{4\,(e)^2}{3} + \frac{(e)^3}{2} + \frac{(e)^4}{24}\right] \quad \textbf{(1.4.2)}$$

> *CPUTime*(*DalgFunGuess*(L, *degADE* = 4, *degPoly* = 0, *startfromord* = 4, *linsolver* = *HardSystem*))

$$12972.390, \_C \left(\frac{d^2}{dx^2} y(x)\right)\left(\frac{d}{dx} y(x)\right)^2 y(x) - \_C \left(\frac{d^3}{dx^3} y(x)\right)\left(\frac{d}{dx} y(x)\right) y(x)^2 \quad \textbf{(1.4.3)}$$

$$+ \_C \left(\frac{d^2}{dx^2} y(x)\right) \left(\frac{d}{dx} y(x)\right) y(x)^2 - \_C \left(\frac{d}{dx} y(x)\right)^4 - \_C \left(\frac{d}{dx} y(x)\right)^2 y(x)^2 - \_C \left(\frac{d}{dx} y(x)\right)^3 y(x) + \_C \left(\frac{d^2}{dx^2} y(x)\right)^2 y(x)^2 = 0$$

> *CPUTime*(*DalgFunGuess*(*L*[1 ..20], *degADE* = 4, *degPoly* = 0, *startfromord* = 4, *allPolyDeg* = *true*, *sparsity* = $\frac{3}{4}$, *maxIteration* = 10))

256.531, *FAIL* **(1.4.4)**

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modDalgFunGuess

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Example 5: OEIS A189281

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> *T* := *readdata*("b189281.txt", *integer*, 2) :

> *L* := *map*(*t*→*t*[2], *T*) :

> *numelems*(*L*)

301 **(2.1.1)**

> *L*[−1] #*the last term in the list*

113720695446706151824501413716513498998133114502443963833651312337879\ **(2.1.2)**
039637872970979509146146227449809162455250958370591226250050872560091\
612675838199513549603266265833964410113293850383206080198084501463791\
813639278933714541638534007985488590334155795914226678549895177524611\
113928303695782615247329221265896779669231508033057980731130731433301\
012663712949384276260257744796080264928953437335628427294873164877701\
777332862303492353547625021840410985380390375278287771210565958912731\
036591949559949054890805401357008975839596499225321849715279543326001\
8184454492026423318241834214846655218599914677589113826427287643819

> *CPUTime*(*modDalgFunGuess*(*L*, *degADE* = 1, *degPoly* = 11, *startfromord* = 19, *modulus* = 569))

$$4.890, \left(132\, \_C\, x^2 + 43\, \_C\, x + 266\, \_C\right) \left(\frac{d^2}{dx^2} y(x)\right) + \left(267\, \_C\, x^3 + 62\, \_C\, x^2 + 309\, \_C\, x + 50\, \_C\right) \left(\frac{d^3}{dx^3} y(x)\right) + \left(138\, \_C\, x^4 + 203\, \_C\, x^3 + 544\, \_C\, x^2\right.$$ **(2.1.3)**

$$+ 329\,\_C\,x + 436\,\_C\Big)\left(\frac{d^4}{dx^4}\,y(x)\right) + \big(118\,\_C\,x^5 + 181\,\_C\,x^4 + 63\,\_C\,x^3$$
$$+ 288\,\_C\,x^2 + 381\,\_C\,x + 444\,\_C\big)\left(\frac{d^5}{dx^5}\,y(x)\right) + \big(555\,\_C\,x^6 + 334\,\_C\,x^5$$
$$+ 475\,\_C\,x^4 + 92\,\_C\,x^3 + 15\,\_C\,x^2 + 37\,\_C\,x + 211\,\_C\big)\left(\frac{d^6}{dx^6}\,y(x)\right)$$
$$+ \big(195\,\_C\,x^7 + 524\,\_C\,x^6 + 294\,\_C\,x^5 + 69\,\_C\,x^4 + 459\,\_C\,x^3 + 179\,\_C\,x^2$$
$$+ 56\,\_C\,x + 195\,\_C\big)\left(\frac{d^7}{dx^7}\,y(x)\right) + \big(337\,\_C\,x^{11} + 165\,\_C\,x^{10} + 256\,\_C\,x^9$$
$$+ 324\,\_C\,x^8 + 473\,\_C\,x^7 + 441\,\_C\,x^6 + 54\,\_C\,x^5 + 189\,\_C\,x^4 + 357\,\_C\,x^3$$
$$+ 395\,\_C\,x^2 + 381\,\_C\,x + 90\,\_C\big)\left(\frac{d^{13}}{dx^{13}}\,y(x)\right) + \big(513\,\_C\,x^{11} + 354\,\_C\,x^{10}$$
$$+ 379\,\_C\,x^9 + 369\,\_C\,x^8 + 306\,\_C\,x^7 + 36\,\_C\,x^6 + 242\,\_C\,x^5 + 307\,\_C\,x^4$$
$$+ 522\,\_C\,x^3 + 222\,\_C\,x^2 + 108\,\_C\,x + 68\,\_C\big)\left(\frac{d^{14}}{dx^{14}}\,y(x)\right) + \big(518\,\_C\,x^{11}$$
$$+ 410\,\_C\,x^{10} + 35\,\_C\,x^9 + 209\,\_C\,x^8 + 23\,\_C\,x^7 + 549\,\_C\,x^6 + 333\,\_C\,x^5$$
$$+ 226\,\_C\,x^4 + 210\,\_C\,x^3 + 509\,\_C\,x^2 + 186\,\_C\,x + 188\,\_C\big)\left(\frac{d^{15}}{dx^{15}}\,y(x)\right)$$
$$+ \big(553\,\_C\,x^{11} + 544\,\_C\,x^{10} + 557\,\_C\,x^9 + 229\,\_C\,x^8 + 315\,\_C\,x^7 + 331\,\_C\,x^6$$
$$+ 376\,\_C\,x^5 + 369\,\_C\,x^4 + 191\,\_C\,x^3 + 6\,\_C\,x^2 + 441\,\_C\,x + 4\,\_C\big)\left(\frac{d^{16}}{dx^{16}}\right.$$
$$\left.y(x)\right) + \big(412\,\_C\,x^8 + 124\,\_C\,x^7 + 340\,\_C\,x^6 + 464\,\_C\,x^5 + 237\,\_C\,x^4$$
$$+ 531\,\_C\,x^3 + 351\,\_C\,x^2 + 283\,\_C\,x + 514\,\_C\big)\left(\frac{d^8}{dx^8}\,y(x)\right) + \big(262\,\_C\,x^9$$
$$+ 535\,\_C\,x^8 + 80\,\_C\,x^7 + 233\,\_C\,x^6 + 248\,\_C\,x^5 + 565\,\_C\,x^4 + 353\,\_C\,x^3$$
$$+ 380\,\_C\,x^2 + 58\,\_C\,x + 426\,\_C\big)\left(\frac{d^9}{dx^9}\,y(x)\right) + \big(522\,\_C\,x^{10} + 85\,\_C\,x^9$$
$$+ 21\,\_C\,x^8 + 136\,\_C\,x^7 + 58\,\_C\,x^6 + 522\,\_C\,x^5 + 263\,\_C\,x^4 + 373\,\_C\,x^3$$
$$+ 565\,\_C\,x + 233\,\_C\big)\left(\frac{d^{10}}{dx^{10}}\,y(x)\right) + \big(447\,\_C\,x^{11} + 458\,\_C\,x^{10} + 57\,\_C\,x^9$$
$$+ 459\,\_C\,x^8 + 320\,\_C\,x^7 + 363\,\_C\,x^6 + 479\,\_C\,x^5 + 451\,\_C\,x^4 + 445\,\_C\,x^3$$
$$+ 362\,\_C\,x^2 + 61\,\_C\,x + 545\,\_C\big)\left(\frac{d^{11}}{dx^{11}}\,y(x)\right) + \big(417\,\_C\,x^{11} + 375\,\_C\,x^{10}$$
$$+ 63\,\_C\,x^9 + 394\,\_C\,x^8 + 35\,\_C\,x^7 + 196\,\_C\,x^6 + 195\,\_C\,x^5 + 426\,\_C\,x^4$$

$$+ 39\_C x^3 + 230\_C x^2 + 154\_C x + 321\_C\right) \left(\frac{d^{12}}{dx^{12}} y(x)\right) + \left(523\_C x^{11}$$

$$+ 375\_C x^{10} + 47\_C x^9 + 280\_C x^8 + 338\_C x^7 + 507\_C x^6 + 278\_C x^5$$

$$+ 233\_C x^4 + 565\_C x^2\right) \left(\frac{d^{17}}{dx^{17}} y(x)\right) + \left(282\_C x^{11} + 467\_C x^{10} + 458\_C x^9$$

$$+ 135\_C x^8 + 432\_C x^7 + 566\_C x^6 + 565\_C x^5\right) \left(\frac{d^{18}}{dx^{18}} y(x)\right) + \left(\_C x^{11}$$

$$+ \_C x^{10} + 568\_C x^9 + 568\_C x^8\right) \left(\frac{d^{19}}{dx^{19}} y(x)\right) = 0$$

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